Caltech – Ph106a – Fall 2001

Problem Set 4 *due 11-14-2001*

1. H&F 3-2 (bead on a rotating hoop), with the following two additional parts added:

   (e) Suppose there is a frictional force, $F_{\text{friction}} = -\gamma v$, where v is the velocity of the bead sliding on the hoop. Show how you would incorporate this into a Lagrangian treatment at the level of the equations of motion. (That is, write the correct equation(s) of motion with this damping force included). Are the equilibrium points affected?

   (f) Use the method of Lagrange multipliers to determine the force exerted by the hoop on the bead. You should neglect friction in this part.

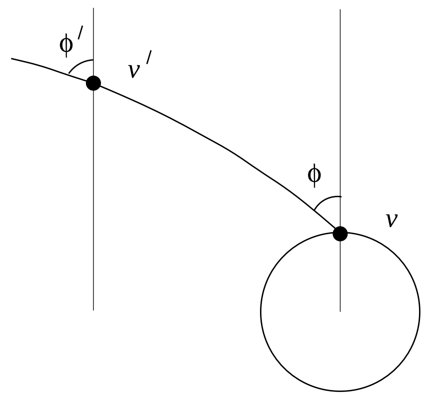
2. H&F 3-16 (driven undamped oscillator).

3. H&F 9-1 (pendulum mounted on a sliding block). Once you have finished the problem as stated in H&F, consider what happens if the block slides with a frictional force $F_{\text{friction}} = -\gamma v$. I claim that the general solution can still be written in a form analogous to what we saw in lecture for undamped oscillations: that is,

$$\vec{q}(t) = \sum_{j=1}^{4} c_j \vec{A}_{(j)} e^{-i\omega_{(j)} t},$$

   where $\vec{q}$ are the generalized coordinates (X and θ, if you like), and the c_j are four complex constants, satisfying constraints that make the whole solution real. Write a polynomial equation that determines the $\omega_{(j)}$ (they are not all real). You need not solve this equation, nor try to obtain the $\vec{A}_{(j)}$.

4. A meteor is observed to strike Earth with a speed v, making an angle ϕ with the zenith (a vertical line drawn through its point of impact). As it approached from far away, the meteor's speed was v' and its direction with respect to the impact zenith was ϕ'. Determine v' and ϕ' in terms of v, ϕ, and known constants.



5. M&T 8-26 (changing orbits) Show that the most efficient way to change the energy of an elliptical orbit for a single short engine thrust is by firing the rocket along the direction of travel at perigee. (Remember, perigee is the point of closest approach to the earth).

6. M&T 9-48 (repulsive $1/r^3$ scattering) A fixed force center scatters a particle of mass m according to the force law $F(r) = k/r^3$. If the initial velocity of the particle is u_0, show that the differential scattering cross section is

$$\frac{d\sigma}{d\Omega} = \frac{k\pi^2(\pi - \theta)}{m u_0^2 \theta^2 (2\pi - \theta)^2 \sin\theta} .$$

7. **Optional challenge problem**
Demonstrate that for any central force law other than $F \propto -1/r^2$ and $F \propto -r$, not all bound orbits are closed. (I put the problem this way because there will almost always be *some* closed orbits, for example circular ones, but what's claimed to be special about gravity and the harmonic oscillator is that *all* orbits close—they're ellipses in both cases).

8. **Optional challenge problem**

   (a) Derive the *range formula* for ballistic missiles. If you launch a missile at an angle γ from vertical and at an initial velocity v_0 (i.e. the velocity after the rocket has stopped firing, which happens soon after launch), then it will travel in an approximately elliptical orbit until it comes back down. A convenient measure of how far the missile goes is the arc-distance Θ_e. (If the whole of the motion is in the equatorial plane, then Θ_e is the change in longitude). Define $\epsilon = v_0^2/(gR_e)$ where R_e is the radius of the earth. Demonstrate that the maximum possible value for Θ_e is

$$\Theta_e = 4\tan^{-1}\frac{1}{\sqrt{1-\epsilon}} - \pi ,$$

and that this value is attained at a launch angle $\gamma = (\Theta_e + \pi)/4$. You may wish to consider the limit where $R_e \to \infty$ in order to check your result against something known.

(b) Use the result from the previous part to determine the minimal velocity needed to get an ICBM from the capital of North Korea (P'yongyang) to LA. (I picked North Korea because of recent concerns in the US about development in this country of a nuclear arsenal and of medium and long-range ballistic missiles).