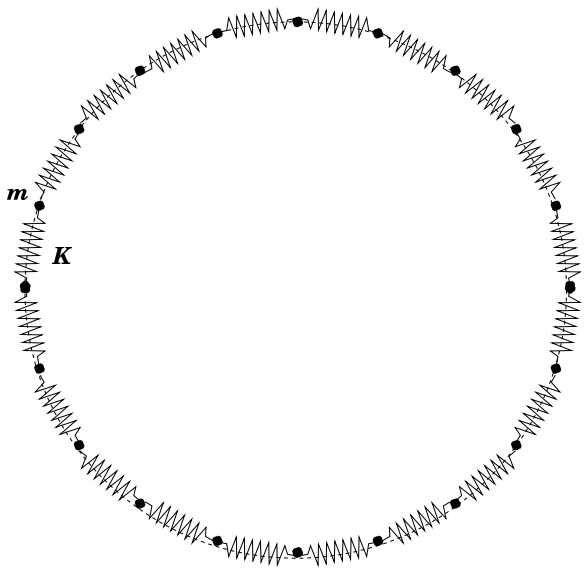
Caltech – Ph106a – Fall 2001

Problem Set 3 *due 10-31-2001*

1. H&F 2-5 (action for QM).

2. (N beads on a hoop) *This problem is long, so it will count for 20 points instead of 10 like the others on this set.*
   Consider N identical masses sliding without friction on a stationary circular hoop whose circumference is Na. Let's connect these masses with springs of spring constant K and natural length a, as shown in the figure. Gravity is negligible. The aim of this problem is to figure out all the normal modes and



   their frequencies. Note that the case $N = 3$ is worked out in Hand and Finch, section 9.6.

   (a) Write down the lagrangian and the equations of motion for this system. Use coordinates q_1, q_2, ..., q_N describing the positions of the particles on the hoop.

   (b) Show that the "plane wave" ansatz, $q_j = \mathrm{Re}\{e^{ikaj - i\omega t}\}$, is a solution to the equations of motion, provided frequency $\omega = \sqrt{K/m}\left|2\sin\frac{ka}{2}\right|$. Here k is supposed to be any real number, but we'll restrict it in the next part of the problem.

   (c) Only certain values of the wave-vector k are permissable. What are they? (*Hint: require* $q_{N+1} = q_1$).

(d) Some values of k are physically indistinguishable from others, in that they result in the same ω and the same $q_j(t)$. What is the criterion for different k to be physically indistinguishable?

(e) How many physically distinguishable values of k are permissable? This should be the number of normal modes of the system. Is it? (*Hint: if you come up one short, think about whether possible degenerate cases.*)

3. (matrices)

(a) H&F 9-16.

(b) H&F 9-17. Also, write the matrix $\begin{pmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{pmatrix}$ in the form $\mathbf{\Lambda}^{-1}\mathbf{D}\mathbf{\Lambda}$, where Λ is orthogonal.

4. (coupled pendula with different masses) H&F 9-8.

5. **Optional challenge problem**
In lecture we noted that in Einstein's model of the specific heat of a solid, the temperature parameter, T_E, came out to be about 1320 K for diamond. See for example Charles Kittel's text on condensed matter physics for a discussion of Einstein's model.

(a) Make a first-principles estimate of T_E, based on the fact that diamond is made of covalently bonded carbon atoms. (Don't kill yourself if your estimate is somewhat off).

(b) Make another estimate based on the fact that the bulk modulus for diamond is roughly 5×10^{11} dynes per square centimeter, and its density is roughly 3.5 grams per cubic centimeter.