

Caltech – Ph106a – Fall 2001

Problem Set 0

due 10-10-2001

The intent of this PS is to be sure we're all on the same page. If you feel your intelligence is being insulted, think of this as 50 free points!

Here and in future problem sets, M&T will denote problems taken from Marion and Thornton's *Classical Dynamics*. I write them out in full, so you need not refer to the text.

1. Matrices and vectors:

(a) (M&T 1-14): Consider

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \\ 2 & 0 & 1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 1 & 3 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \\ 1 & 0 \end{pmatrix}.$$

Compute a) $|\mathbf{AB}|$; b) \mathbf{AC} ; c) \mathbf{ABC} ; d) $\mathbf{AB} - \mathbf{A}^T\mathbf{B}^T$, where T means transpose.

(b) (M&T 1-27): If \vec{r} and $\dot{\vec{r}} = \vec{v}$ are both explicit functions of time, show that

$$\frac{d}{dt}[\vec{r} \times (\vec{v} \times \vec{r})] = r^2\vec{a} + (\vec{r} \cdot \vec{v})\vec{v} - (v^2 + \vec{r} \cdot \vec{a})\vec{r}.$$

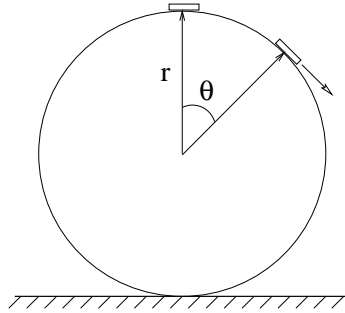
2. Freshman physics:

(a) (M&T 2-5): A jet fighter pilot knows he is able to withstand an acceleration of $9g$ before blacking out. The pilot points his plane vertically down while traveling at Mach 3 speed and intends to pull up in a circular maneuver before crashing into the ground. a) Where does the maximum acceleration occur in the maneuver? b) What is the minimum circle the pilot can take?

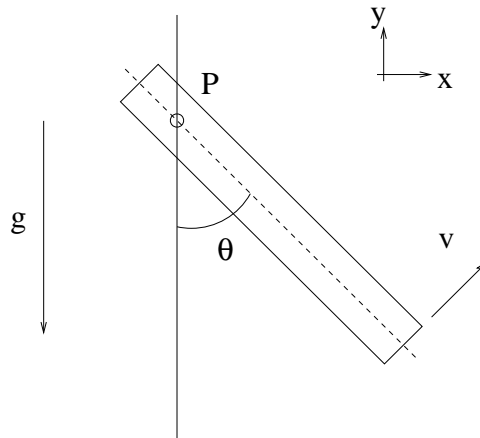
(b) (M&T 2-14): A projectile is fired with initial speed v_0 at an elevation angle of α up a hill of slope $\beta < \alpha$. a) How far up the hill will the projectile land? b) At what angle α will the range be a maximum? c) What is the maximum range?

(c) (M&T 2-27): A rope having a total mass of 0.4 kg and total length 4 m has 0.6 m of the rope hanging vertically down off a work bench. How much work must be done to place all the rope on the bench?

3. (puck on a sphere) A puck, initially at rest on top of a smooth sphere of radius r , is given an infinitesimal push to the right. The puck slides freely without friction. At what angle will the puck leave the sphere?



4. (compound pendulum) A compound pendulum is formed by a uniform thin bar of mass m and length l that rotates on a pivot P , located $3/4$ of the way from its lower end.



- Find the equation of motion for the pendulum, assuming that the angle θ between the pendulum and the vertical remains small.
 - Assuming the pendulum is released from rest from an initial angle θ_0 , find an expression $\theta(t)$ for the pendulum's subsequent motion.
 - Find a vector expression $\mathbf{v}(t)$ (*i.e.* find v_x and v_y) for the velocity of the lower end of the pendulum given the same initial conditions as in part (b).
5. **Optional Challenge Problem.**
 A uniform thin rod of mass m and length l hangs in static equilibrium, suspended by two *ideal* strings. The string at the left is cut. What is the acceleration of the left end of the rod, the moment after the string is cut? Now, instead of ideal strings, what happens when this is done with *real strings* (or wires)? What is the initial acceleration of the left end of the rod now?

(Hint: an ideal string amounts to a exact constraint: one end can't be further from the other than the length of the string. A real string might still be negligibly

massive as compared to the rod, but inevitably it would have some extensibility, like a spring.)

